

SECTION A

$20 \times 1 = 20$ marks

This section has 18 multiple-choice questions and 2 assertion–reason questions of 1 mark each. For questions 19 and 20, choose the correct option (A), (B), (C) or (D) given below each question.

1. The range of the function $\pi/3 - \tan^{-1} x$ is:

- A $[-\frac{\pi}{6}, \frac{5\pi}{6}]$
- B $(-\frac{2\pi}{3}, \frac{\pi}{3})$
- C $(\frac{\pi}{3}, \frac{4\pi}{3})$
- D $(-\frac{\pi}{6}, \frac{5\pi}{6})$

2. The domain of the function $\sin^{-1}((\frac{3}{2})^x)$ is:

- A $(\frac{-2}{3}, \frac{2}{3})$
- B $[\frac{-2}{3}, \frac{2}{3}]$
- C $[\frac{-3}{2}, \frac{3}{2}]$
- D $(-\infty, \infty)$

3. The value of $\cot^{-1}(\sin(2 \sin^{-1}(1/\sqrt{2})))$ is:

- A $\frac{\pi}{3}$
- B $\frac{\pi}{6}$
- C $\frac{\pi}{2}$
- D $\frac{\pi}{4}$

THE WORKED ANSWERS TO THOSE 3 QUESTIONS

1. The range of the function $\pi/3 - \tan^{-1} x$ is: [Inverse trigonometric functions]

Correct answer: D — $(-\frac{\pi}{6}, \frac{5\pi}{6})$

- The principal value range of $\tan^{-1} x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- $\pi/3 - \tan^{-1} x$ therefore takes values from $-\pi/6$ to $5\pi/6$.
- The range is $(-\frac{\pi}{6}, \frac{5\pi}{6})$.
- Why the other options are wrong:
 - A $([-\pi/6, 5\pi/6])$: Use the principal value range of $\tan^{-1} x$ and shift it by $\pi/3$.
 - B $((-2\pi/3, \pi/3))$: Use the principal value range of $\tan^{-1} x$ and shift it by $\pi/3$.
 - C $((\pi/3, 4\pi/3))$: Use the principal value range of $\tan^{-1} x$ and shift it by $\pi/3$.

LOS: Relations and Functions

KEY TAKEAWAY

Shift (and flip) the principal value range of the inverse function.

2. The domain of the function $\sin^{-1}(\frac{3}{2}x)$ is: [Inverse trigonometric functions]

Correct answer: B — $[-\frac{2}{3}, \frac{2}{3}]$

- The domain of $\sin^{-1}$ is $[-1, 1]$.
- $|\text{argument}| \leq 1$ gives x between $-\frac{2}{3}$ and $\frac{2}{3}$, including the end points.
- Why the other options are wrong:
 - A $([-\frac{2}{3}, \frac{2}{3}))$: The domain comes from $|\text{argument}| \leq 1$; check the end points and the size of k .
 - C $([-\frac{3}{2}, \frac{3}{2}])$: The domain comes from $|\text{argument}| \leq 1$; check the end points and the size of k .
 - D $((-\infty, \infty))$: The domain comes from $|\text{argument}| \leq 1$; check the end points and the size of k .

LOS: Relations and Functions

KEY TAKEAWAY

Domains: $\sin^{-1}$ and $\cos^{-1}$ need $|u| \leq 1$; $\sec^{-1}$ and cosec^{-1} need $|u| \geq 1$.

3. The value of $\cot^{-1}(\sin(2 \sin^{-1}(1/\sqrt{2})))$ is: [Inverse trigonometric functions]

Correct answer: D — $\frac{\pi}{4}$

- $\sin^{-1}(1/\sqrt{2}) = \frac{\pi}{4}$.
- The inner angle times 2 is $\frac{\pi}{2}$, and sin of that is 1.
- $\cot^{-1}$ of 1 = $\frac{\pi}{4}$.
- Why the other options are wrong:
- A ($\pi/3$): Work from the inside out: $\sin^{-1}(1/\sqrt{2})$ first, then the sin, then the outer inverse function.
- B ($\pi/6$): Work from the inside out: $\sin^{-1}(1/\sqrt{2})$ first, then the sin, then the outer inverse function.
- C ($\pi/2$): Work from the inside out: $\sin^{-1}(1/\sqrt{2})$ first, then the sin, then the outer inverse function.

LOS: Relations and Functions

KEY TAKEAWAY

Evaluate inverse trigonometric compositions from the inside out using standard angles.