

JEE ADVANCED — PAPER 1 — 48 QUESTIONS

Mathematics · Physics · Chemistry · 180 marks · 3 hours

Work through each subject in turn. Read the marking scheme at the head of every section: wrong answers lose marks in Sections 1, 2 and 4.

Mathematics — Section 1: one correct option

4 questions · four options, only ONE is correct · Correct +3, wrong or marked in any other way –1, unanswered 0 · Maximum marks 12

1. The number of terms in the expansion of $(2^{\frac{1}{3}} + 3^{\frac{1}{4}})^{72}$ that are integers is

- A 4
- B 5
- C 7
- D 14

2. A function $f: \mathbb{N} \rightarrow \mathbb{N}$ satisfies $f(x + y) = f(x) + f(y) + 3xy$ for all natural numbers x, y , and $f(1) = 2$. The value of $f(13)$ is

- A 260
- B 286
- C 288
- D 520

3. The tangent at the point $P(18, 12)$ of the parabola $y^2 = 8x$ meets the x -axis at T . If F is the focus, the area of triangle PTF is

- A 96
- B 120
- C 126
- D 240

THE WORKED ANSWERS TO THOSE 3 QUESTIONS

1. The number of terms in the expansion of $(2^{\frac{1}{3}} + 3^{\frac{1}{4}})^{72}$ that are integers is [Binomial theorem]

Correct answer: C — 7

- The general term is $C(72, k) \cdot 2^{\frac{k}{3}} \cdot 3^{\frac{72-k}{4}}$.
- It is an integer when k is a multiple of 3 and $(72 - k)$ is a multiple of 4.
- Counting $k = 0..72$ satisfying both gives 7.
- Why the other options are wrong:
- A (4): This is 3 less than the correct value (7); check the arithmetic of the last step.
- B (5): This is 2 less than the correct value (7); check the arithmetic of the last step.
- D (14): This is twice the correct value (7): a factor of 2 has been added.

LOS: Mathematics · Binomial theorem

KEY TAKEAWAY

Terms with integer powers of both radicals: solve the two congruences on k together.

2. A function $f: \mathbb{N} \rightarrow \mathbb{N}$ satisfies $f(x + y) = f(x) + f(y) + 3xy$ for all natural numbers x, y , and $f(1) = 2$. The value of $f(13)$ is [Functions]

Correct answer: A — 260

- Put $y = 1$: $f(x + 1) = f(x) + f(1) + 3x = f(x) + 2 + 3x$.
- Summing for $x = 1, \dots, 12$: $f(13) = 13 \cdot 2 + 3(1 + 2 + \dots + 12)$.
- $= 26 + 3 \cdot 78 = 260$.
- Why the other options are wrong:
- B (286): This is about 1.1 times the correct value (260); it does not follow from the steps above.
- C (288): This is about 1.1 times the correct value (260); it does not follow from the steps above.
- D (520): This is twice the correct value (260): a factor of 2 has been added.

LOS: Mathematics · Functions

KEY TAKEAWAY

Substitute $y = 1$ to turn a functional equation into a recurrence, then sum it.

3. The tangent at the point $P(18, 12)$ of the parabola $y^2 = 8x$ meets the x -axis at T . If F is the focus, the area of triangle PTF is [Conic sections]

Correct answer: B — 120

- For $y^2 = 4ax$ the tangent at $(at^2, 2at)$ is $ty = x + at^2$; with $a = 2$, $t = 3$ it meets $y = 0$ at $T(-18, 0)$.
- The focus is $F(2, 0)$, so $TF = 2 + 18 = 20$.
- Height of P above the x -axis $= 12$; area $= \frac{1}{2} \times 20 \times 12 = 120$.
- Why the other options are wrong:
- A (96): This is only about 0.8 of the correct value (120); it does not follow from the steps above.
- C (126): This is 6 more than the correct value (120); check the arithmetic of the last step.
- D (240): Forgot the factor $\frac{1}{2}$ in $\frac{1}{2} \times \text{base} \times \text{height}$.

LOS: Mathematics · Conic sections

KEY TAKEAWAY

The tangent to $y^2 = 4ax$ at $(at^2, 2at)$ is $ty = x + at^2$; it meets the axis at $(-at^2, 0)$.