

JEE ADVANCED — PAPER 2 — 48 QUESTIONS

Mathematics · Physics · Chemistry · 180 marks · 3 hours

Work through each subject in turn. Read the marking scheme at the head of every section: wrong answers lose marks in Sections 1, 2 and 4.

Mathematics — Section 1: one correct option

4 questions · four options, only ONE is correct · Correct +3, wrong or marked in any other way –1, unanswered 0 · Maximum marks 12

1. The number of distinct arrangements of all the letters of the word DIVISION in which the vowels appear in the order in which they occur in the word (as a subsequence) is

- A 1680
- B 1764
- C 2520
- D 6720

2. L_1 is the line through the point $(-1, -2, 3)$ with direction vector $(0, 0, -3)$, and L_2 is the line through $(-4, -3, 0)$ with direction vector $(0, -3, -4)$. The shortest distance between L_1 and L_2 is

- A 1
- B 2
- C 3
- D 27

3. If $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^2 x \, dx = p/q$ in lowest terms, then $p + q$ is

- A 16
- B 248
- C 315
- D 331

THE WORKED ANSWERS TO THOSE 3 QUESTIONS

1. The number of distinct arrangements of all the letters of the word DIVISION in which the vowels appear in the order in which they occur in the word (as a subsequence) is
[Permutations and combinations]

Correct answer: A — 1680

- Choose which 4 of the 8 places hold vowels: $C(8, 4) = 70$.
- The vowels fill those places in the fixed order, so there is one way for them.
- Arrange the consonants in the other places: 24 ways.
- Why the other options are wrong:
- B (1764): This is about 1.1 times the correct value (1680); it does not follow from the steps above.
- C (2520): This is about 1.5 times the correct value (1680); it does not follow from the steps above.
- D (6720): Counted all arrangements of the letters, ignoring the condition.

LOS: Mathematics · Permutations and combinations

KEY TAKEAWAY

Place the unrestricted letters first, then fit the restricted ones into gaps or fixed order.

2. L_1 is the line through the point $(-1, -2, 3)$ with direction vector $(0, 0, -3)$, and L_2 is the line through $(-4, -3, 0)$ with direction vector $(0, -3, -4)$. The shortest distance between L_1 and L_2 is
[Three-dimensional geometry]

Correct answer: C — 3

- $d_1 \times d_2 = (-9, 0, 0)$, with magnitude 9.
- Joining the given points: $w = (-3, -1, -3)$; $w \cdot (d_1 \times d_2) = 27$.
- Distance = $|w \cdot (d_1 \times d_2)| / |d_1 \times d_2| = \frac{27}{9} = 3$.
- Why the other options are wrong:
- A (1): This is one third of the correct value (3).
- B (2): This is 1 less than the correct value (3); check the arithmetic of the last step.
- D (27): Forgot to divide by $|d_1 \times d_2|$.

LOS: Mathematics · Three-dimensional geometry

KEY TAKEAWAY

Distance between skew lines = $|(a_2 - a_1) \cdot (d_1 \times d_2)| / |d_1 \times d_2|$.

3. If $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^2 x \, dx = p/q$ in lowest terms, then $p + q$ is [Integral calculus]

Correct answer: D — 331

- Use the Wallis-type formula $\int_0^{\frac{\pi}{2}} \sin^a x \cos^b x \, dx = [(a-1)!!(b-1)!!]/(a+b)!! \times (\frac{\pi}{2})$ if a and b are both even, otherwise 1).
- For $a = 7$, $b = 2$ the value is $\frac{16}{315}$.
- So $p + q = 16 + 315 = 331$.
- Why the other options are wrong:
- A (16): Gave p only.
- B (248): This is only about 0.75 of the correct value (331); it does not follow from the steps above.
- C (315): Gave q only.

LOS: Mathematics · Integral calculus

KEY TAKEAWAY

Wallis-type integrals: $\int_0^{\frac{\pi}{2}} \sin^a x \cos^b x$ is a ratio of double factorials, with an extra $\frac{\pi}{2}$ only when both a and b are even.