

FREE SAMPLE: THE FIRST 3 QUESTIONS

Question 1 (30 marks)

(a) Solve the following equation in x . Give each answer correct to 2 decimal places.

$$5x^2 + 14x - 6 = 0$$

[10 marks]

(b) Find the range of values of $p \in \mathbb{R}$ for which:

$$|3p + 1| \geq 9$$

[10 marks]

(c) Solve the equation for $x \in \mathbb{R}$:

$$\log_3(x + 20) + \log_3(x - 6) = 3$$

[10 marks]

Question 2 (30 marks)

(a) Prove by induction that the following identity is true for all $n \in \mathbb{N}$:

$$1 + 3 + 9 + \dots + 3^n = \frac{3n+1-1}{2}$$

[10 marks]

(b) Solve for $x \in \mathbb{R}$:

$$5^x = 125^{x+3}$$

[10 marks]

(c) A geometric sequence has $T_4 = 108$ and $T_5 = 324$, and a common ratio that is an integer.

- [10 marks]

Question 3 (30 marks)

- (ii) Use De Moivre's theorem to find z^2 , giving your answer in the form $p + qi$, where $p, q \in \mathbb{R}$.

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THE WORKED ANSWERS TO THOSE 3 QUESTIONS

Question 1 (30 marks) [Algebra and equations]

(a) Solve the following equation in x . Give each answer correct to 2 decimal places. $5x^2 + 14x - 6 = 0$

(a) Answer: $x \approx -3.18$ or $x \approx 0.38$

- Compare with $ax^2 + bx + c = 0$: $a = 5$, $b = 14$, $c = -6$.
- Substitute into $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \frac{-14 \pm \sqrt{(14)^2 - 4(5)(-6)}}{10}$.
- The discriminant is $b^2 - 4ac = 316$, so $\sqrt{316} \approx 17.7764$.
- $x = \frac{-14 + 17.7764}{10} \approx 0.38$ and $x = \frac{-14 - 17.7764}{10} \approx -3.18$.

KEY TAKEAWAY

Quadratic formula: write a , b , c carefully with their signs, check the discriminant is positive, keep four or more decimals until the final rounding to 2 d.p.

(b) Find the range of values of $p \in \mathbb{R}$ for which: $|3p + 1| \geq 9$

(b) Answer: $p \leq \frac{-10}{3}$ or $p \geq \frac{8}{3}$

- $|A| \geq k$ means $A \geq k$ or $A \leq -k$, so $3p + 1 \geq 9$ or $3p + 1 \leq -9$.
- Solve each linear inequality: $p \geq \frac{8}{3}$ and $p \leq \frac{-10}{3}$.
- Combine the two outer ranges: $p \leq \frac{-10}{3}$ or $p \geq \frac{8}{3}$.

KEY TAKEAWAY

Modulus inequalities: $|A| \geq k$ splits into two outer ranges (or), $|A| \leq k$ gives one inner range (and); divide by a positive number without flipping the sign.

(c) Solve the equation for $x \in \mathbb{R}$: $\log_3(x + 20) + \log_3(x - 6) = 3$

(c) Answer: $x = 7$

- Both arguments must be positive, so $x > 6$ (the larger of -20 and 6).
- Add the logs as a single log of a product: $\log_3((x + 20)(x - 6)) = 3$, so $(x + 20)(x - 6) = 3^3 = 27$.
- Expand and collect: $x^2 + 14x - 147 = 0$.
- Factorise: $(x - 7)(x + 21) = 0$, so $x = 7$ or $x = -21$.
- $x = 7$ is greater than 6, so it is valid. $x = -21$ is less than 6, so it is rejected. Therefore $x = 7$.

KEY TAKEAWAY

Adding logs of the same base multiplies the arguments; after solving the quadratic, test each root against the domain of every log.

LOS: Algebra and equations (Leaving Certificate Mathematics, Strand 4 Algebra)

Question 2 (30 marks) [Proof, indices and sequences]

(a) Prove by induction that the following identity is true for all $n \in \mathbb{N}$: $1 + 3 + 9 + \dots + 3^n = \frac{3n+1-1}{2}$

(a) Answer: Proof by induction completed (base case, assumption for k, step to k + 1, conclusion).

- Base case $n = 1$: here the sum is $1 + 3 = 4$. $\text{RHS} = \frac{3^2 - 1}{2} = 4$. True for $n = 1$.
- Assume true for $n = k$: $1 + 3 + \dots + 3^k = \frac{3^{k+1} - 1}{2}$.
- Add the next term 3^{k+1} : $\text{LHS} = \frac{3^{k+1} - 1}{2} + 3^{k+1} = \frac{3^{k+1} - 1 + 2 \cdot 3^{k+1} - 3^{k+1}}{2} = \frac{3^{k+2} - 1}{2}$.
- This is $\frac{3^{n+1} - 1}{2}$ with $n = k + 1$, so the statement holds for $n = k + 1$.
- True for $n = 1$, and true for $k + 1$ whenever true for k , so by induction it is true for all $n \in \mathbb{N}$.

KEY TAKEAWAY

Induction: state the base case, write the assumption for $n = k$, use it to reach the expression for $n = k + 1$, and close with the concluding sentence.

(b) Solve for $x \in \mathbb{R}$: $5^x = 125^{x+3}$

(b) Answer: $x = \frac{-9}{2}$

- Write both sides with the same base 5: $5 = 5^1$ and $125 = 5^3$.
- So $(5^1)^x = (5^3)^{x+3}$, which gives $5^x = 5^{3(x+3)}$.
- Equal bases, so equal indices: $x = 3(x + 3) = 3x + 9$.
- Collect x terms: $-2x = 9$, so $x = \frac{-9}{2}$.

KEY TAKEAWAY

Exponential equations: write every power with the same prime base, equate the indices, then solve the linear equation.

(c) A geometric sequence has $T_4 = 108$ and $T_5 = 324$, and a common ratio that is an integer. (i) Find the first term a and the common ratio r . (ii) Find the sum of the first 6 terms.

(c) Answer: $a = 4$, $r = 3$; $S_6 = 1456$

- $T_n = ar^{n-1}$, so $ar^3 = 108$ and $ar^4 = 324$.
- Divide the terms: $r = \frac{324}{108} = 3$. Then $a = 108 / 3^3 = 4$.
- $S_n = a(r^n - 1)/(r - 1)$, so $S_6 = 4 \frac{3^6 - 1}{3 - 1} = 1456$.

KEY TAKEAWAY

Geometric sequences: divide two terms to get a power of r , take the root, then use $S_n = a(r^n - 1)/(r - 1)$.

LOS: Proof, indices and sequences (Leaving Certificate Mathematics, Strands 3 and 4)

Question 3 (30 marks) [Complex numbers]

(a) Let $z = \sqrt{3} - i$. (i) Write z in polar form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give θ in radians as a multiple of π . (ii) Use De Moivre's theorem to find z^2 , giving your answer in the form $p + qi$, where $p, q \in \mathbb{R}$.

(a) Answer: (i) $z = 2(\cos(\frac{-\pi}{6}) + i \sin(\frac{-\pi}{6}))$; (ii) $z^2 = 2 - (2\sqrt{3})i$

- (i) $r = |z| = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$.
- z lies in the fourth quadrant and $\tan \theta = b/a = \frac{-1}{\sqrt{3}}$, so $\theta = \frac{-\pi}{6}$.
- So $z = 2(\cos(\frac{-\pi}{6}) + i \sin(\frac{-\pi}{6}))$.
- (ii) De Moivre: $z^2 = 2^2 (\cos(\frac{-\pi}{3}) + i \sin(\frac{-\pi}{3})) = 4(\cos(\frac{-\pi}{3}) + i \sin(\frac{-\pi}{3}))$.
- Evaluate: $z^2 = 2 - (2\sqrt{3})i$.

KEY TAKEAWAY

Polar form: $r = \sqrt{a^2 + b^2}$, find θ from the quadrant (not the calculator alone); De Moivre multiplies the angle by n and raises r to the power n .

(b) Use De Moivre's theorem to solve the equation $w^3 = 27$. Give each answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

(b) Answer: $w = 3, \frac{-3}{2} + (3\sqrt{\frac{3}{2}})i, \frac{-3}{2} - (3\sqrt{\frac{3}{2}})i$

- Write the right-hand side in polar form: $27 = 27(\cos(0) + i \sin(0))$; more generally add $2\pi k$ to the angle.
- Let $w = r(\cos \phi + i \sin \phi)$. De Moivre: $w^3 = r^3(\cos 3\phi + i \sin 3\phi)$, so $r^3 = 27$ and $3\phi = 0 + 2\pi k$.
- Hence $r = 3$ and $\phi = \frac{0 + 2\pi k}{3}$ for $k = 0, 1, \dots, 2$.
- $k = 0$: $w = 3$.
- $k = 1$: $w = \frac{-3}{2} + (3\sqrt{\frac{3}{2}})i$.
- $k = 2$: $w = \frac{-3}{2} - (3\sqrt{\frac{3}{2}})i$.

KEY TAKEAWAY

De Moivre roots: an n th-root equation has exactly n solutions, spaced $\frac{2\pi}{n}$ apart on a circle of radius $r^{1/n}$; add $2\pi k$ to the angle before dividing by n .

(c) $z = (2 + i)$ is a root of the equation: $z^3 - 5z^2 + 9z - 5 = 0$. Find the other two roots of this equation. One should be complex and the other should be real.

(c) Answer: $z = 2 - i$ and $z = 1$

- The coefficients of the polynomial are real, so complex roots come in conjugate pairs. The conjugate $2 - i$ is also a root.
- A quadratic factor is $(z - (2 + i))(z - (2 - i)) = z^2 - 4z + 5$.
- Divide the cubic by this quadratic: the quotient is the linear factor $(z - 1)$.
- Check using the constant term: the product of the roots is 5, and $(2 + i)(2 - i)(1) = 5 \times 1 = 5$.
- The other two roots are $z = 2 - i$ (complex) and $z = 1$ (real).

KEY TAKEAWAY

Conjugate root theorem: a polynomial with real coefficients has complex roots in conjugate pairs; multiply the pair to get a real quadratic factor, then divide to find the remaining root.