

[10 marks]

(b) The line ℓ has equation $x - y + 4 = 0$. The point R has co-ordinates $(8, 0)$.

Find the shortest distance from R to the line ℓ . Give your answer in the form p/q , where $p, q \in \mathbb{N}$.

[10 marks]

(a) The three points $A(-1, 2)$, $B(6, 9)$ and $C(2, -5)$ form a triangle. θ is the acute angle between $[AB]$ and $[AC]$.

- (i) Find the slope of the line AB and the slope of the line AC.
- (ii) Hence, or otherwise, find the value of θ . Give your answer correct to 1 decimal place.

[10 marks]

- (b) The line ℓ has equation $x - y + 4 = 0$. The point R has co-ordinates $(8, 0)$.

Find the shortest distance from R to the line ℓ . Give your answer in the form p/q , where $p, q \in \mathbb{Z}$.

[10 marks]

(c) The line ℓ has equation $5x + 3y - 11 = 0$. The point P has co-ordinates $(-3, -2)$.

Find the equation of the line through P that is perpendicular to ℓ . Give your answer in the form $px + qy + r = 0$, where $p, q, r \in \mathbb{Z}$.

[10 marks]

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THE WORKED ANSWER TO THAT QUESTION

Question 1 (30 marks) [Coordinate geometry of the line]

(a) The three points A(-1, 2), B(6, 9) and C(2, -5) form a triangle. θ is the acute angle between [AB] and [AC]. (i) Find the slope of the line AB and the slope of the line AC. (ii) Hence, or otherwise, find the value of θ . Give your answer correct to 1 decimal place.

(a) Answer: (i) slope AB = 1, slope AC = $-\frac{7}{3}$; (ii) $\theta \approx 68.2^\circ$

- (i) Slope = $\frac{y_2 - y_1}{x_2 - x_1}$. Slope of AB = $(9 - 2)/(6 - (-1)) = 1$.
- Slope of AC = $(-5 - 2)/(2 - (-1)) = -\frac{7}{3}$.
- (ii) Use $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| (1 - (-\frac{7}{3})) / (1 + (1)(-\frac{7}{3})) \right| = \frac{5}{2}$.
- $\theta = \tan^{-1}(2.5000) = 68.2^\circ$.

KEY TAKEAWAY

Angle between two lines: find the two slopes, then use $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ (from the Formulae and Tables) and take $\tan^{-1}$.

(b) The line ℓ has equation $x - y + 4 = 0$. The point R has co-ordinates (8, 0). Find the shortest distance from R to the line ℓ . Give your answer in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$.

(b) Answer: $d = 6\sqrt{2}$

- The shortest distance is the perpendicular distance: $d = |ax_1 + by_1 + c| / \sqrt{a^2 + b^2}$, with $a = 1$, $b = -1$, $c = 4$ and $(x_1, y_1) = (8, 0)$.
- Numerator: $|1(8) + (-1)(0) + 4| = |12| = 12$.
- Denominator: $\sqrt{(1^2 + (-1)^2)} = \sqrt{2}$.
- $d = 12 / (1/\sqrt{2}) = 12\sqrt{2} / 2 = 6\sqrt{2}$.

KEY TAKEAWAY

Distance from a point to a line: use $|ax_1 + by_1 + c| / \sqrt{a^2 + b^2}$, then rationalise the denominator to reach the form $p\sqrt{q}$.

(c) The line ℓ has equation $5x + 3y - 11 = 0$. The point P has co-ordinates (-3, -2). Find the equation of the line through P that is perpendicular to ℓ . Give your answer in the form $px + qy + r = 0$, where $p, q, r \in \mathbb{Z}$.

(c) Answer: $3x - 5y - 1 = 0$

- Slope of $\ell = \frac{-a}{b} = \frac{-5}{3}$.
- The perpendicular slope is the negative reciprocal: $\frac{3}{5}$.
- Use $y - y_1 = m(x - x_1)$ through (-3, -2): $y - (-2) = 3/5(x - (-3))$.
- Multiply by 5 and rearrange to get $3x - 5y - 1 = 0$.

KEY TAKEAWAY

Perpendicular lines: slopes multiply to -1 (negative reciprocal); then use $y - y_1 = m(x - x_1)$ and tidy to integer coefficients.

LOS: Coordinate geometry of the line (Leaving Certificate Mathematics, Higher Level Paper 2, Strand 2 Geometry)