FREE SAMPLE: THE FIRST QUESTION

Question One (8 marks)

(a) A stone is launched at 18.0 m s^{-1} at 35.0° above the horizontal from the edge of a cliff, 22.0 m above level ground. Ignore air resistance. Calculate the time of flight and the horizontal distance travelled before it lands.

[2 marks]

(b) Find the speed and the angle below the horizontal of the velocity at the instant of landing. Show that this speed would be the same for any launch angle.

[3 marks]

(c) It is given that, for a launch from height h at speed u, the horizontal range is greatest when $\tan\theta = u \div \sqrt{(u^2 + 2gh)}$.

Find this angle and the greatest range for the data above, and show that a launch at 60.0° gives a shorter range.

[3 marks]

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THE WORKED ANSWER TO THAT QUESTION

Question 1 (8 marks) [Projectile motion from a height]

(a) A stone is launched at 18.0 m s^{-1} at 35.0° above the horizontal from the edge of a cliff, 22.0 m above level ground. Ignore air resistance. Calculate the time of flight and the horizontal distance travelled before it lands.

(a) Answer: T = 3.42 s; R = 50.4 m

- Entry step: worth marks in the 1-4 range of the 8-point question score.
- Take up as positive: initial vertical speed 18.0 sin 35.0° = 10.32 m s^{-1} and horizontal speed 18.0 cos 35.0° = 14.74 m s^{-1}. Landing: $-22.0 = 10.32\,t - \tfrac{1}{2}(9.81)t^2$. [1 mark]
- Solve the quadratic and take the positive root: t = 3.417 s. Horizontal distance = 14.74 × 3.417 = 50.39 m. [1 mark]

> **KEY TAKEAWAY**
> Split the motion into independent vertical and horizontal parts; the landing condition gives a quadratic in time and only the positive root is physical.

(b) Find the speed and the angle below the horizontal of the velocity at the instant of landing. Show that this speed would be the same for any launch angle.

(b) Answer: v = 27.5 m s^{-1} at 57.6° below the horizontal

- Scholarship band (5-6): the part needs your own method and accurate algebra.
- Vertical speed at landing = 10.32 − 9.81(3.417) = −23.20 m s^{-1} (downwards). Speed = 27.49 m s^{-1}. [1 mark]
- Angle = $\tan^{-1}(23.20 \div 14.74)$ = 57.56° below the horizontal. [1 mark]
- Energy conservation: $\tfrac{1}{2}mv^2 = \tfrac{1}{2}mu^2 + mgh$, so $v = \sqrt{(u^2 + 2gh)}$ depends only on u and h, not on the launch angle. [1 mark]

> **KEY TAKEAWAY**
> A conservative force changes speed by an amount that depends only on the height change, so the landing speed does not depend on the launch angle.

(c) It is given that, for a launch from height h at speed u, the horizontal range is greatest when $\tan\theta = u \div \sqrt{(u^2 + 2gh)}$. Find this angle and the greatest range for the data above, and show that a launch at 60.0° gives a shorter range.

(c) Answer: θ = 33.2°; Rmax = 50.4 m (at 60.0° the range is 38.1 m)

- Outstanding band (7-8): the part needs insight and a complete, efficient argument.
- $\tan\theta = 18.0 \div 27.49 = 0.6548$ so θ = 33.22°. [1 mark]
- At this angle, $R_{max} = u\sqrt{(u^2 + 2gh)} \div g = 18.0 \times 27.49 \div 9.81 = 50.44$ m. [1 mark]
- At 60.0°: T = 4.237 s and R = 18.0 × cos 60.0° × 4.237 = 38.13 m, which is less than 50.44 m. [1 mark]

> **KEY TAKEAWAY**
> For a launch from above the landing level the best angle is below 45°; a quick calculation at a second angle confirms the optimum is a maximum.

LOS: Projectile motion from a height (93103)

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