

## FREE SAMPLE: THE FIRST 3 QUESTIONS

### PAPER 1

Pure Mathematics · 100 marks · Questions 1 – 11 · 3 hours

Answer ALL questions. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified. You are expected to use a graphing calculator (GC). Unsupported answers from a GC are allowed unless a question states otherwise; where a question says 'show' or 'find the exact', present the mathematical steps. The last question applies mathematics to a real-world context. The number of marks is given in brackets at the end of each part.

#### Question 1 (6 marks)

- (a) Find  $\int (e^{4x} + \frac{2}{4x+1}) dx$ . [2 marks]

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Answer \_\_\_\_\_

- (b) Using integration by parts, find the exact value of  $\int$  from 0 to  $\frac{\pi}{2}$  of  $x \cos(x) dx$ . [4 marks]

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Answer \_\_\_\_\_

#### Question 2 (8 marks)

A sequence  $u_1, u_2, u_3, \dots$  is given by  $u_1 = 4$  and  $u_{n+1} = \frac{2}{3} u_n + 4$ .

- (a) Find  $u_2$  and  $u_3$  as fractions or integers. [2 marks]

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Answer \_\_\_\_\_

(b) The sequence converges to a limit  $L$ . Find  $L$ . [2 marks]

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**Answer** \_\_\_\_\_

(c) Given that  $u_n - L = (u_1 - L)(2/3)^{n-1}$ , find the least value of  $n$  for which  $|u_n - L| < 0.001$ . [4 marks]

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**Answer** \_\_\_\_\_

### Question 3 (8 marks)

A curve satisfies the differential equation  $dy/dx = 1/3(y - 3)$ .

(a) Find the general solution of the differential equation. [3 marks]

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**Answer** \_\_\_\_\_

(b) Given that  $y = 1$  when  $x = 0$ , find the particular solution. [2 marks]

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**Answer** \_\_\_\_\_

(c) Find the exact value of  $x$  for which  $y = \frac{7}{3}$ . [3 marks]

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**Answer** \_\_\_\_\_

## THE WORKED ANSWERS TO THOSE 3 QUESTIONS

### Question 1 (6 marks) [Integration techniques]

(a) Find  $\int (e^{4x} + \frac{2}{4x+1}) dx$ .

**(a) Answer:**  $(\frac{1}{4})e^{4x} + (\frac{2}{4})\ln|4x+1| + C$

- $\int e^{4x} dx = (\frac{1}{4})e^{4x}$ .
- $\int \frac{2}{4x+1} dx = (\frac{2}{4})\ln|4x+1|$ .
- $(\frac{1}{4})e^{4x} + (\frac{2}{4})\ln|4x+1| + C$

(b) Using integration by parts, find the exact value of  $\int$  from 0 to  $\frac{\pi}{2}$  of  $x \cos(x) dx$ .

**(b) Answer:**  $-1 + \frac{\pi}{2}$

- Choose  $u$  and  $dv/dx$  so that differentiating  $u$  simplifies the integrand, then  $\int u dv = uv - \int v du$ .
- Apply the formula (repeat if needed) and substitute the limits 0 to  $\frac{\pi}{2}$ .
- The exact value is  $-1 + \frac{\pi}{2}$ .

LOS: Pure Mathematics · Integration techniques

#### KEY TAKEAWAY

Choose parts so the differentiated factor gets simpler; with a substitution always change the limits as well as the integrand.

### Question 2 (8 marks) [Sequences and series]

A sequence  $u_1, u_2, u_3, \dots$  is given by  $u_1 = 4$  and  $u_{n+1} = \frac{2}{3}u_n + 4$ .

(a) Find  $u_2$  and  $u_3$  as fractions or integers.

**(a) Answer:**  $u_2 = \frac{20}{3}, u_3 = \frac{76}{9}$

- $u_2 = \frac{2}{3} \times 4 + 4 = \frac{20}{3}$ .
- $u_3 = \frac{2}{3} \times \frac{20}{3} + 4 = \frac{76}{9}$ .

(b) The sequence converges to a limit  $L$ . Find  $L$ .

**(b) Answer:**  $L = 12$

- As  $n \rightarrow \infty$ ,  $u_{n+1}$  and  $u_n$  both tend to  $L$ , so  $L = \frac{2}{3}L + 4$ .
- $L(1 - \frac{2}{3}) = 4$ , so  $L = 12$ .

(c) Given that  $u_n - L = (u_1 - L)(\frac{2}{3})^{n-1}$ , find the least value of  $n$  for which  $|u_n - L| < 0.001$ .

**(c) Answer:**  $n = 24$

- $|u_1 - L| = |4 - 12| = 8$ .
- Require  $8 \times |\frac{2}{3}|^{n-1} < 1/1000$ ; use logarithms or the table on the GC.
- The least  $n$  is 24.

LOS: Pure Mathematics · Sequences and series

**KEY TAKEAWAY**

For a recurrence  $u_{n+1} = f(u_n)$ , the limit  $L$  satisfies  $L = f(L)$ .

**Question 3 (8 marks) [Differential equations]**

A curve satisfies the differential equation  $dy/dx = 1/3(y - 3)$ .

(a) Find the general solution of the differential equation.

**(a) Answer:  $y = 3 + Ae^{(x/3)}$**

- Separate the variables:  $\int \frac{1}{y-3} dy = \int \frac{1}{3} dx$ .
- $\ln|y - 3| = \frac{x}{3} + C$ , so  $y - 3 = Ae^{(x/3)}$  where  $A = \pm e^C$ .
- Hence  $y = 3 + Ae^{(x/3)}$ .

(b) Given that  $y = 1$  when  $x = 0$ , find the particular solution.

**(b) Answer:  $y = 3 - 2e^{(x/3)}$**

- Substitute  $x = 0, y = 1$ :  $1 = 3 + A$ , so  $A = -2$ .
- $y = 3 - 2e^{(x/3)}$ .

(c) Find the exact value of  $x$  for which  $y = \frac{7}{3}$ .

**(c) Answer:  $x = -\ln(27)$**

- Solve  $\frac{7}{3} = 3 - 2e^{(x/3)}$ :  $e^{(x/3)} = \frac{1}{3}$ .
- Take natural logarithms:  $x = -\ln(27)$ .

LOS: Pure Mathematics · Differential equations

**KEY TAKEAWAY**

Separate the variables, integrate both sides, then use the initial condition to fix the constant.