

FREE SAMPLE: THE FIRST 3 QUESTIONS

PAPER 1

90 marks · Questions 1 – 13 · 2 h 15 min

Answer ALL questions. Omission of essential working will result in loss of marks. A calculator may be used. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified. Use the calculator value of π , or 3.142, unless the answer is required in terms of π . The number of marks is given in brackets at the end of each part.

Question 1 (5 marks)

The polynomial $f(x) = x^3 + ax^2 + bx - 8$ is such that $(x - 1)$ is a factor and the remainder is 0 when $f(x)$ is divided by $(x + 2)$.

(a) Find the values of a and b .

Answer _____

Question 2 (5 marks)

(a) Express $6\cos \theta + \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and α is acute. Give the exact value of R and the value of α correct to 1 decimal place.

Answer _____

Question 3 (6 marks)

The curve C has equation $y = 2x^2 + 10x - 2$ and the line L has equation $y = 2x + k$, where k is a constant.

- (a) Show that the x -coordinates of the points where L meets C satisfy the equation $2x^2 + 8x + (-2 - k) = 0$. [2 marks]

Answer _____

- (b) Given that L is a tangent to C, find the value of k . [2 marks]

Answer _____

- (c) Find the coordinates of the point where L touches C. [2 marks]

Answer _____

THE WORKED ANSWERS TO THOSE 3 QUESTIONS

Question 1 (5 marks) [Polynomials and partial fractions]

The polynomial $f(x) = x^3 + ax^2 + bx - 8$ is such that $(x - 1)$ is a factor and the remainder is 0 when $f(x)$ is divided by $(x + 2)$.

(a) Find the values of a and b .

(a) Answer: $a = 5$, $b = 2$

- Factor theorem: $f(1) = 0$, so $1 + a(1) + b(1) - 8 = 0$.
- Remainder theorem: $f(-2) = 0$, so $-8 + a(4) + b(-2) - 8 = 0$.
- Solve the two simultaneous equations: $a = 5$ and $b = 2$.

LOS: Algebra · Polynomials and partial fractions

KEY TAKEAWAY

The factor theorem $f(p) = 0$ and the remainder theorem $f(p) = R$ turn conditions into linear equations for the unknown coefficients.

Question 2 (5 marks) [Trigonometric functions, identities and equations]

(a) Express $6\cos \theta + \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and α is acute. Give the exact value of R and the value of α correct to 1 decimal place.

(a) Answer: $R = \sqrt{37}$, $\alpha = 9.5^\circ$

- $R \cos(\theta - \alpha) = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$, so $R \cos \alpha = 6$ and $R \sin \alpha = 1$.
- $R = \sqrt{6^2 + 1^2} = \sqrt{37}$ and $\tan \alpha = \frac{1}{6}$, so $\alpha = 9.5^\circ$.

LOS: Geometry and Trigonometry · Trigonometric functions, identities and equations

KEY TAKEAWAY

Writing a $\cos \theta + b \sin \theta$ as $R \cos(\theta \pm \alpha)$ turns a two-term expression into a single wave, making maximum values and equations easy.

Question 3 (6 marks) [Equations and inequalities]

The curve C has equation $y = 2x^2 + 10x - 2$ and the line L has equation $y = 2x + k$, where k is a constant.

(a) Show that the x -coordinates of the points where L meets C satisfy the equation $2x^2 + 8x + (-2 - k) = 0$.

(a) Answer: $2x^2 + 8x + (-2 - k) = 0$

- At a point of intersection the y -values are equal: $2x^2 + 10x - 2 = 2x + k$.
- Rearranging gives $2x^2 + 8x + (-2 - k) = 0$.

(b) Given that L is a tangent to C, find the value of k.

(b) Answer: $k = -10$

- A tangent meets the curve at one point only, so the discriminant is zero: $(8)^2 - 4(2)(-2 - k) = 0$.
- $(8)^2 = 8(-2 - k)$, so $k = -2 - 8$.
- $k = -10$.

(c) Find the coordinates of the point where L touches C.

(c) Answer: $(-2, -14)$

- For equal roots $x = -\frac{b-m}{2a} = -\frac{8}{4} = -2$.
- $y = 2(-2) - 10 = -14$.

LOS: Algebra · Equations and inequalities

KEY TAKEAWAY

A line is a tangent to a curve when substituting it gives a quadratic with discriminant zero; positive discriminant means two intersections, negative means none.